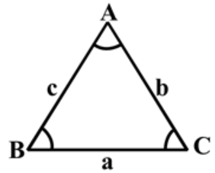
MIND MAP

Solution of Triangle

1. Properties & Solution of Triangle



When three elements of a triangle are given, the process of calculating its other three elements is called SOLUTION OF TRIANGLE.

In any triangle ABC,

Sum of two sides of a triangle is always greater than the third side.

$a + b > c$ $b + c > a$ $c + a > b$

Difference of any two sides of a triangle is less than the third side.

$|a - c| < b$ $|b - a| < c$ & $|c - b| < a$

2. Sine Rule

In any ΔABC, sines of the angles are proportional to the opposite sides.

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = K}$$

3. Cosine Rule

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}} \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\boxed{\cos B = \frac{c^2 + a^2 - b^2}{2ca}} \quad b^2 = c^2 + a^2 - 2ca \cos B$$

$$\boxed{\cos C = \frac{a^2 + b^2 - c^2}{2ab}} \quad c^2 = a^2 + b^2 - 2ab \cos C$$

4. Projection Formula

$$\boxed{a = b \cos C + c \cos B} \quad \boxed{b = c \cos A + a \cos C}$$

$$\boxed{c = a \cos B + b \cos A}$$

5. Tangent Rule

$$\boxed{\tan \frac{B - C}{2} = \frac{b - c}{b + c} \cot \frac{A}{2}}$$

$$\boxed{\tan \frac{C - A}{2} = \frac{c - a}{c + a} \cot \frac{B}{2}}$$

$$\boxed{\tan \frac{A - B}{2} = \frac{a - b}{a + b} \cot \frac{C}{2}}$$

6. Half Angle Formulae

$$s = \frac{a + b + c}{2} \text{(Semi Perimeter)}$$

$$\sin \frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{bc}} \qquad \cos \frac{A}{2} = \sqrt{\frac{s(s - a)}{bc}}$$

$$\sin \frac{B}{2} = \sqrt{\frac{(s - c)(s - a)}{ca}} \qquad \cos \frac{B}{2} = \sqrt{\frac{s(s - b)}{ca}}$$

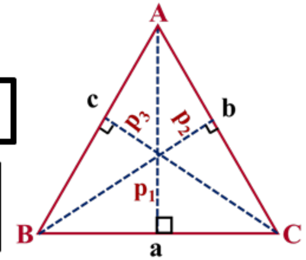
$$\sin \frac{C}{2} = \sqrt{\frac{(s - a)(s - b)}{ab}} \qquad \cos \frac{C}{2} = \sqrt{\frac{s(s - c)}{ab}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}} = \frac{\Delta}{s(s - a)}$$

$$\tan \frac{B}{2} = \sqrt{\frac{(s - c)(s - a)}{s(s - b)}} = \frac{\Delta}{s(s - b)}$$

$$\tan \frac{C}{2} = \sqrt{\frac{(s - a)(s - b)}{s(s - c)}} = \frac{\Delta}{s(s - c)}$$

7. Area of the Triangle

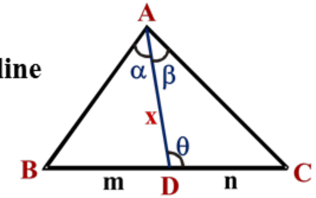


$$\boxed{\Delta = \sqrt{s(s - a)(s - b)(s - c)}}$$

$$\boxed{\Delta = \frac{1}{2} a p_1 = \frac{1}{2} b p_2 = \frac{1}{2} c p_3}$$

$$\boxed{\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C}$$

8. m-n Theorem (Cotangent Law)



If in a ΔABC,

D is a point on the line BC such that

$BD : DC = m : n$

$\angle ADC = \theta$, $\angle BAD = \alpha$, $\angle DAC = \beta$, then

$$\boxed{\text{(a) } (m + n) \cot \theta = m \cot \alpha - n \cot \beta}$$

$$\boxed{\text{(b) } (m + n) \cot \theta = n \cot B - m \cot C}$$

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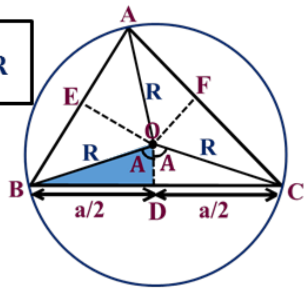
9. Circles connected with Triangle

Circumcircle of a Triangle

The circle passing through the vertices of a triangle is called the circumcircle of the triangle.

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R}$$

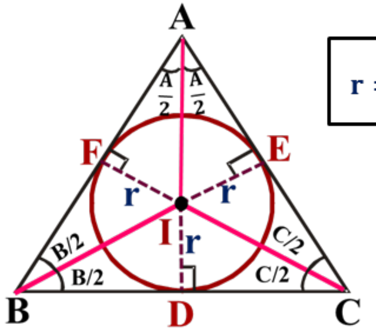
$$\boxed{R = \frac{abc}{4\Delta} = \frac{a}{2\sin A} = \ldots}$$



Distance of circumcentre from vertices is R and from sides is $R\cos A$, $R\cos B$ & $R\cos C$.

For a right angled triangle, circumcentre is the mid point of hypotenuse

Incircle (Inscribed Circle) of a Triangle



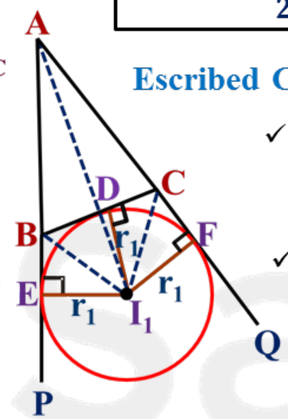
$$\boxed{r = \frac{\Delta}{s}}$$

$$\boxed{r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}}$$

Relation b/w inradius & circumradius

$$\boxed{r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$$

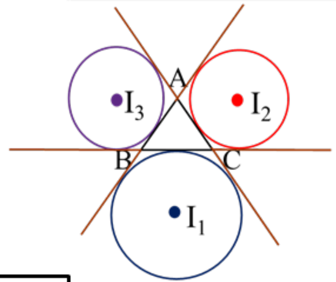
Escribed Circles of A Triangle



✓ The circle is called escribed circle or Ex-circle to ΔABC.

✓ Excentre is the point where two exterior angle bisectors & one internal angle bisector meet.

✓ Centres of the escribed circles → excentres.



$r_1 \rightarrow$ radius of E_1

$r_2 \rightarrow$ radius of E_2

$r_3 \rightarrow$ radius of E_3

($r_1, r_2, r_3 \rightarrow$ exradii)

$$\boxed{r_1 = \frac{\Delta}{s-a}\ ;\ r_2 = \frac{\Delta}{s-b}\ ;\ r_3 = \frac{\Delta}{s-c}}$$

Relation b/w Exradii & Circumradius

$$r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$$

$$r_3 = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

	A	B	C
I_1	$r_1 \operatorname{cosec}\frac{A}{2}$	$r_1 \sec\frac{B}{2}$	$r_1 \sec\frac{C}{2}$
I_2	$r_2 \sec\frac{A}{2}$	$r_2 \operatorname{cosec}\frac{B}{2}$	$r_2 \sec\frac{C}{2}$
I_3	$r_3 \sec\frac{A}{2}$	$r_3 \sec\frac{B}{2}$	$r_3 \operatorname{cosec}\frac{C}{2}$

10. Orthocentre

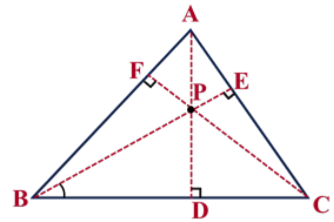
Point of intersection of three altitudes is called Orthocentre

Distances of the Orthocentre from Vertices & Sides of ΔABC

$PA = 2R|\cos A|$ $\quad P_{AB} = 2R|\cos A \cos B|$

$PB = 2R|\cos B|$ $\quad P_{BC} = 2R|\cos B \cos C|$

$PC = 2R|\cos C|$ $\quad P_{CA} = 2R|\cos C \cos A|$



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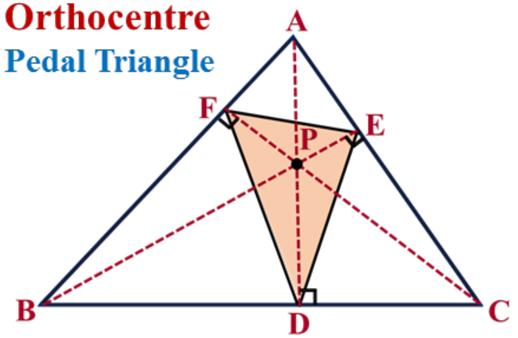
10. Orthocentre

Pedal Triangle



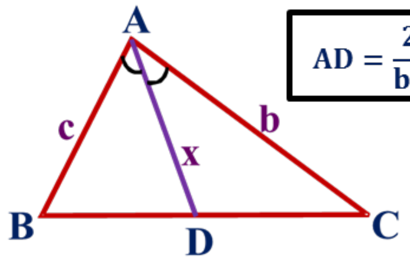
Sides & Angles of Pedal Δ

EF = R sin2A = a cosA $\angle D = \pi - 2A$

DE = R sin2C = c cosC $\angle E = \pi - 2B$

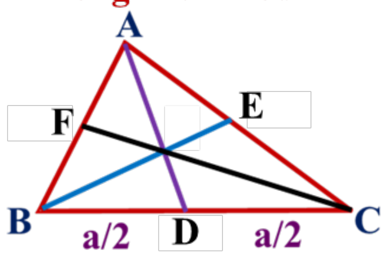
FD = R sin2B = b cosB $\angle F = \pi - 2C$

11. Length of Angle Bisector



$$AD = \frac{2bc}{b+c}\cos\frac{A}{2}$$

12. Length of Median



$$AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

$$BE = \frac{1}{2}\sqrt{2c^2 + 2a^2 - b^2}$$

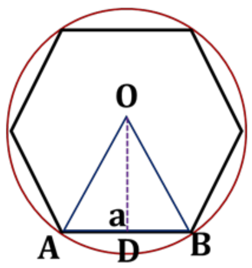
$$CF = \frac{1}{2}\sqrt{2c^2 + 2a^2 - b^2}$$

Note : If ℓ_1, ℓ_2, ℓ_3 are the lengths of the three medians, then $4(\ell_1^2 + \ell_2^2 + \ell_3^2) = 3(a^2 + b^2 + c^2)$

12. Regular Polygon

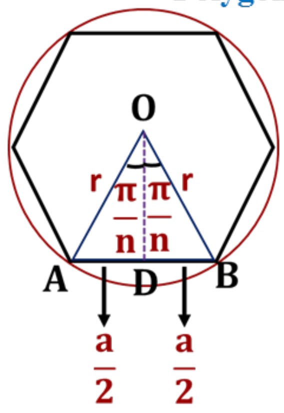
A regular polygon has all it's sides as well as all it's angles equal.

If the polygon has 'n' sides, sum of it's internal angles is $(n-2)\pi$ and each angle is $\frac{(n-2)\pi}{n}$.



$$\angle AOD = \frac{\pi}{n}$$

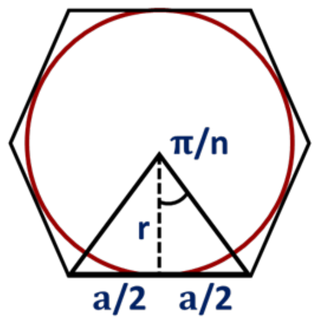
Perimeter And Area of A Regular Polygon of n Sides Inscribed In a Circle of Radius r



Perimeter : $= 2nr\sin\frac{\pi}{n}$

Area : $= \frac{1}{2}n.\, r^2 \sin\left(\frac{2\pi}{n}\right)$

Perimeter and Area of a Regular Polygon of n Sides Circumscribed about a Circle of Radius r



Perimeter $= 2nr\tan\frac{\pi}{n}$

Area $= n\frac{1}{2}a.\, r = \frac{n}{2}2r^2\tan\frac{\pi}{n} = nr^2\tan\frac{\pi}{n}$